

Hong Kong Mathematics Olympiad (2007 – 2008)

Heat Event (Group)

香港數學競賽 (2007 – 2008)

初賽項目(團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 已知 $5 + \sqrt{11}$ 的小數部分為 A 及 $5 - \sqrt{11}$ 的小數部分為 B 。設 $C = A + B$ ，求 C 的值。

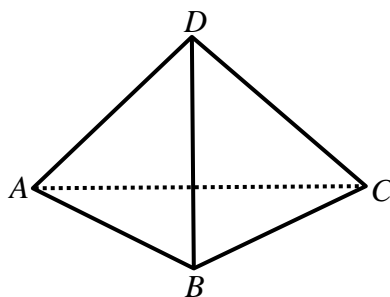
Given that the decimal part of $5 + \sqrt{11}$ is A and the decimal part of $5 - \sqrt{11}$ is B . Let $C = A + B$, find the value of C .

2. 有一批糖共 x 粒， x 為正整數，這批糖能分別為 851 人及 943 人所均分。求 x 的最小可能值。

A total number of x candies, x is a positive integer, can be evenly distributed to 851 people as well as 943 people. Find the least possible value of x .

3. 如圖一，正四面體 $ABCD$ 的邊長為 2 cm。若該正四面體的體積是 $\sqrt{R} \text{ cm}^3$ ，求 R 的值。

In Figure 1, $ABCD$ is a tetrahedron with side length 2 cm. If the volume of the tetrahedron is $\sqrt{R} \text{ cm}^3$, find the value of R .



圖一

Figure 1

4. 已知 x 為正整數及 $x < 60$ 。若 x 恰有 10 個正因子，求 x 的值。

Given that x is a positive integer and $x < 60$. If x has exactly 10 positive factors, find the value of x .

5. 已知 $90^\circ < \theta < 180^\circ$ 及 $\sin \theta = \frac{\sqrt{3}}{2}$ 。若 $A = \cos(180^\circ - \theta)$ ，求 A 的值。

Given that $90^\circ < \theta < 180^\circ$ and $\sin \theta = \frac{\sqrt{3}}{2}$. If $A = \cos(180^\circ - \theta)$, find the value of A .

6. 設 x 為正實數，求 $x^{2008} - x^{1004} + \frac{1}{x^{1004}}$ 的最小值。

Let x be a positive real number. Find the minimum value of $x^{2008} - x^{1004} + \frac{1}{x^{1004}}$.

7. 設 x 及 y 為實數，且滿足

$$\begin{cases} \left(x - \frac{1}{3}\right)^3 + 2008\left(x - \frac{1}{3}\right) = -5, \\ \left(y - \frac{7}{4}\right)^3 + 2008\left(y - \frac{7}{4}\right) = 5. \end{cases}$$

若 $z = x + y$ ，求 z 的值。

Let x and y be real numbers satisfying

$$\begin{cases} \left(x - \frac{1}{3}\right)^3 + 2008\left(x - \frac{1}{3}\right) = -5, \\ \left(y - \frac{7}{4}\right)^3 + 2008\left(y - \frac{7}{4}\right) = 5. \end{cases}$$

If $z = x + y$, find the value of z .

8. 設 R 為 $1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times 15 \times 17 \times 19$ 被 4 除後的餘數，求 R 的值。

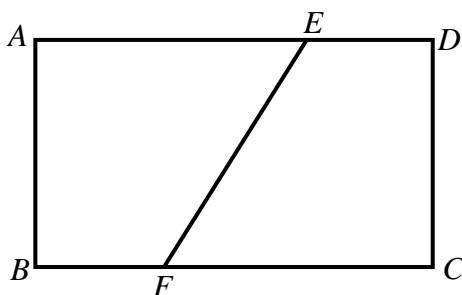
Let R be the remainder of $1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times 15 \times 17 \times 19$ divided by 4. Find the value of R .

9. 已知 k 、 x_1 及 x_2 為正整數且 $x_1 < x_2$ 。設 A 、 B 及 C 為曲線 $y = kx^2$ 上的三點，其 x 坐標分別為 $-x_1$ 、 x_1 及 x_2 。若 $\triangle ABC$ 的面積是 15 平方單位，求所有可能 k 值的總和。

Given that k , x_1 and x_2 are positive integers with $x_1 < x_2$. Let A , B and C be three points on the curve $y = kx^2$, with x coordinates being $-x_1$, x_1 and x_2 respectively. If the area of $\triangle ABC$ is 15 square units, find the sum of all possible values of k .

10. 如圖二， $ABCD$ 是長方形紙張並有 $AB = 4$ cm 及 $BC = 5$ cm。將該紙張對摺，使 C 點與 A 點重合，得摺痕 EF 。若 $EF = r$ cm，求 r 的值。

In Figure 2, $ABCD$ is a rectangular piece of paper with $AB = 4$ cm and $BC = 5$ cm. Fold the paper by putting point C onto A to create a crease EF . If $EF = r$ cm, find the value of r .



圖二

Figure 2